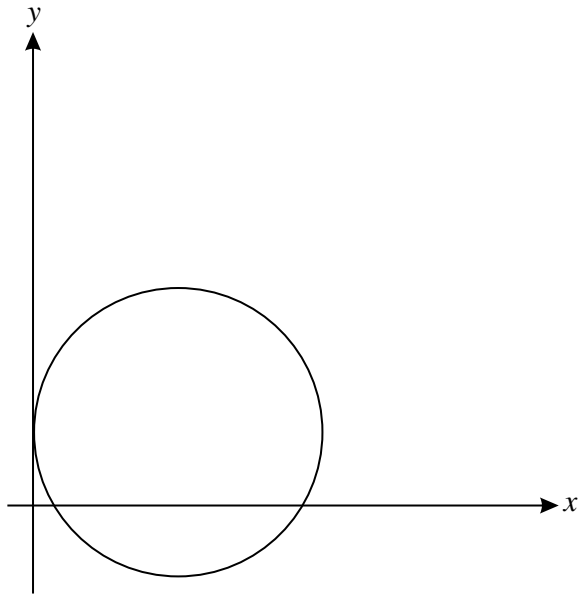**1**

Fig. 13

Fig. 13 shows the circle with equation $(x - 4)^2 + (y - 2)^2 = 16$.

(i) Write down the radius of the circle and the coordinates of its centre. **[2]**

(ii) Find the x-coordinates of the points where the circle crosses the x-axis. Give your answers in surd form. **[4]**

(iii) Show that the point $A\left(4 + 2\sqrt{2},\ 2 + 2\sqrt{2}\right)$ lies on the circle and mark point A on the copy of Fig. 13.

Sketch the tangent to the circle at A and the other tangent that is parallel to it.

Find the equations of both these tangents. **[7]**

2   A circle has equation $(x - 3)^2 + (y + 2)^2 = 25$.

   (i) State the coordinates of the centre of this circle and its radius. **[2]**

   (ii) Verify that the point A with coordinates $(6, -6)$ lies on this circle. Show also that the point B on the circle for which AB is a diameter has coordinates $(0, 2)$. **[3]**

   (iii) Find the equation of the tangent to the circle at A. **[4]**

   (iv) A second circle touches the original circle at A. Its radius is 10 and its centre is at C, where BAC is a straight line. Find the coordinates of C and hence write down the equation of this second circle. **[3]**

3 The curve with equation $y = \frac{1}{5}x(10 - x)$ is used to model the arch of a bridge over a road, where x and y are distances in metres, with the origin as shown in Fig. 12.1. The x-axis represents the road surface.
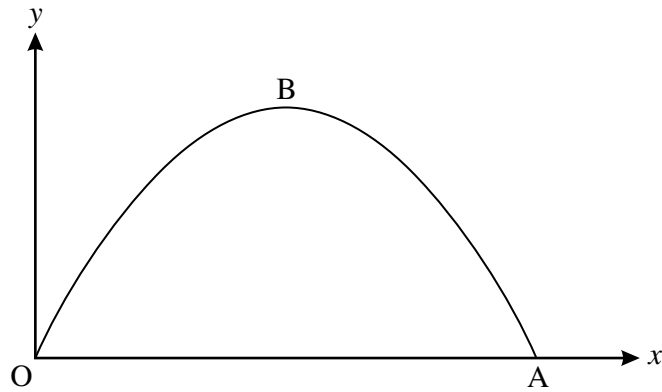

Fig. 12.1

  (i) State the value of x at A, where the arch meets the road. **[1]**

 (ii) Using symmetry, or otherwise, state the value of x at the maximum point B of the graph.

     Hence find the height of the arch. **[2]**

(iii) Fig. 12.2 shows a lorry which is 4 m high and 3 m wide, with its cross-section modelled as a rectangle. Find the value of d when the lorry is in the centre of the road. Hence show that the lorry can pass through this arch. **[3]**
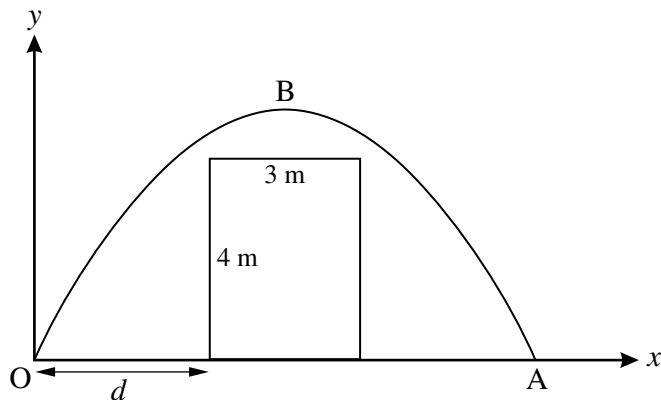

Fig. 12.2

(iv) Another lorry, also modelled as having a rectangular cross-section, has height 4.5 m and just touches the arch when it is in the centre of the road. Find the width of this lorry, giving your answer in surd form. **[5]**

4 A circle has equation $(x - 5)^2 + (y - 2)^2 = 20$.

   (i) State the coordinates of the centre and the radius of this circle. **[2]**

  (ii) State, with a reason, whether or not this circle intersects the y-axis. **[2]**

 (iii) Find the equation of the line parallel to the line $y = 2x$ that passes through the centre of the circle. **[2]**

 (iv) Show that the line $y = 2x + 2$ is a tangent to the circle. State the coordinates of the point of contact. **[5]**